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PREDICTION OF THE STABILITY OF POOL BOILING HEAT TRANSFER TO FINITE DISTURBANCES

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Abstract—A theoretical study has been made of the pool boiling heat-transfer stability on a non-isothermal surface of the rod. The mathematical formulation of the problem is based on the thermal approach, which has allowed the stability study to be performed on the solution of the boundary-value heat-conduction problem for a rod with the boundary conditions of the first and the second kind. The stability to the finite temperature deviations has been analyzed. With the help of a variational principle a measure of stability is found which is a specially built functional. The suggested method is illustrated by calculation of the safe temperature distrubance.

NOMENCLATURE

a, thermal diffusivity of the wall material;
c, specific heat of the wall material;
F, distribution function for normal law;
$J[\theta_{\text{dist.}}]$, value of the functional of a disturbed profile;
$J_{w.sh.}$, value of the functional "at the watershed";
L, rod length;
M, mathematical expectation;
m, number of disturbances for the period $\Delta\tau$;
N, integer;
n, number of events;
$P(n, \Delta\tau)$, Poisson function;
$q(\theta)$, boiling heat flux density removed in line with Nukiyama's boiling curve;
$q_{\text{boil.}}$, heat flux transported by boiling liquid;
$q_{\text{sup.}}$, heat flux supplied to the heating surface;
q_v, power of inner heat sources constant along the coordinate and in time per unit heat-transfer surface area;
s, cross-sectional area of rod;
T, wall temperature;
T_s, saturation temperature;
u, rod cross-section perimeter;
z, coordinate along rod axis.

Greek symbols

α_0, α_L, heat-transfer coefficients at the boundaries $z = 0$ and $z = L$, respectively;
γ_i, constants equal to 0 or 1, whose combinations yield the boundary conditions of the first, second or third kinds ($i = 1, 2, \ldots, 6$);
$\delta\theta$, disturbance of steady-state temperature profile;
$\delta\theta_{cr}$, critical deviation of temperature resulting in one steady-state regime being replaced by another;
θ, excessive surface temperature, $= T - T_s$;
θ_0, θ_L, coolant temperature at the boundaries $z = 0$ and $z = L$, respectively, for the third boundary-value problem;
$\theta_{0_{\text{dist.}}}, \theta_{L_{\text{dist.}}}$, temperature at the rod ends $z = 0$ and $z = L$, respectively, subsequent to disturbance $\delta\theta$;
$\theta_{\text{dist.}}$, temperature profile of the rod subsequent to disturbance $\delta\theta$;
λ, thermal conductivity of wall material;
μ_0, minimum eigenvalue in the Sturm–Liuoville problem [11];
ρ, density of wall material;
σ, standard deviation of random variable $\delta\theta$;
τ, time;
$\tau_{l.t.}$, life-time of the system;
$\tau_{op.}$, time of the setup operation;
$\Delta\tau$, $= \tau_{op.}/N$;
$\chi(\tau)$, parameter of the flux of disturbances of a sample temperature field;
Ω_i, boundaries of stability to infinitesimal disturbances ($i = 1, 2, 3, 4, 5$).

1. INTRODUCTION

As is known, the boiling regime is unambiguously controlled by the temperature of the heating surface. Stability of boiling on the surface of a sufficiently large heating element can be conceived as stability of the temperature field (or of a lengthwise temperature profile when the problem is reduced to one-dimensional) to various deviations of the regime parameters. This approach (we shall call it a thermal one) makes it possible to estimate the heat-transfer stability of infinitesimal and finite temperature deviations for boiling of liquid on both isothermal and non-isothermal heating surfaces. The thermal approach relies on the assigned boiling curve. Construction of this curve is based upon the available correlations for the heat-transfer coefficient. The critical heat fluxes are predicted by the hydrodynamic theory of the boiling crises developed by Kutateladze [1] and Zuber [2].

The problems of boiling stability based on the thermal approach have received attention in the works of many authors. Thus, Stefan [3], considering steady boiling on an isothermal solid surface, defined the instability as a tendency of infinitesimal wall temperature deviations to increase with time. With the Liapunov's first approach he established the condition for the heat-transfer stability as

$$\left(\frac{\partial q_{\text{boil}}}{\partial T}\right) > \left(\frac{\partial q_{\text{sup}}}{\partial T}\right).$$

Using it, Stefan worked out and theoretically justified the method of stabilization of a transient boiling regime.

Employing the same method of small disturbances, Nishikava and Honda [4] obtained the stability conditions for a flat wall, and Hale and Wallis [5], for flat and cylindrical walls.

Adiutori [6] based his analysis of the possible breakdown in the boiling stability on heat-balance grounds.

Treating theoretically the problem of boiling heat-transfer stability on an isothermal surface, all of the above authors have derived a single experimentally confirmed criterion, which indicates the fulfilment of the heat-balance requirement.

As to the same problem but on a non-isothermal surface, this has been studied insufficiently. In [7] a one-dimensional heat conduction equation was suggested for a lengthwise non-isothermal rod with boiling occurring on its lateral side. The author analysed the behaviour of its solution for occasional small deviations of the surface temperature and presented the technique to calculate the magnitude of deviations which lead to a change in the boiling regime. A similar analysis, but based on slightly different premises, was made in [8].

As is seen from the above, the available works (except [7] and [8]) are concerned with the heat-transfer stability to infinitesimal temperature disturbances in boiling on an isothermal surface. Meanwhile, in most cases arising in practice one has to deal with non-isothermal surfaces and the finite temperature deviations. Mathematically, the situation is much more complicated. Therefore, a more detailed physical statement of this problem and its strict mathematical solution have become imminent. These are the objects of the present paper.

2. STATEMENT OF THE PROBLEM

The problem is formulated on the following assumptions:

1. The heat-transfer instability in liquid boiling is due to the breakdown in the heat balance of a solid–liquid system.

2. The mechanism of heat transfer in a boiling liquid is not considered, but the liquid is ascribed with the property of removing heat from the heating wall by a certain prespecified law, known as a "boiling curve". This allows the stability analysis in a conjugate problem to be replaced by that in the heat conduction problem for a solid body.

3. The calculation is based on the experimental boiling curve obtained in the steady-state conditions on an isothermal surface. According to [9], the effect of the surface non-isothermity on heat transfer is insignificant.

4. Such disturbances of the regime parameters are analysed, which are accompanied by the temperature profile deviations.

Consider a rod of the length L, with liquid boiling on its lateral side and heat being supplied through its end faces (heat can also be generated inside the rod itself due to chemical/nuclear reactions or the current passage). If the radial heat flux is small as compared to the axial one and the cross-section mean temperature slightly differs from the true temperature at any point of this cross-section, then heat transfer between the rod and liquid is described by the following boundary-value problem

$$\frac{\partial\theta}{\partial\tau} = a\frac{\partial^2\theta}{\partial z^2} + \frac{u/s}{c\rho}q_v - \frac{u/s}{c\rho}q(\theta), \tag{1}$$

$$\gamma_1\lambda\frac{\partial\theta}{\partial z}\bigg|_{z=0} = \gamma_2\alpha_0\theta(0) - \gamma_3\alpha_0\theta_{0_c}, \tag{2}$$

$$\gamma_4\lambda\frac{\partial\theta}{\partial z}\bigg|_{z=1} = -\gamma_5\alpha_L\theta(L) - \gamma_6\alpha_L\theta_{L_c}, \tag{3}$$

$$\theta(0, z) = \bar{\theta}(z). \tag{4}$$

Solution of this problem is not the aim of the present paper. For, in fact, all the processes in boiling equipment usually occur in a steady-state regime [$\partial\theta/\partial\tau = 0$ in equation (1)], which can be easily described numerically or even analytically. The specific feature of this problem is that, by virtue of the unique form of $q(\theta)$ suggested by Nukiyama, the steady-state solutions to the problem may not be single-valued, i.e. two or more steady temperature distributions correspond to the same boundary conditions. This can be illustrated by the following examples. It is shown in [10], that for boiling on a non-isothermal surface of the fins there is an S-shaped curve of the heat flux q_0 on the heated end of the rod (at $z = 0$) vs the local (at $z = 0$) excess temperature, $\theta_0 = T_0 - T_s$ (Fig. 1, curve 1, the other end of the rod, $z = L$, is adiabatically insulated). If on the heated end the first kind boundary conditions are adopted, i.e. a temperature, $\theta_{0_{1,2,3}}$, is kept constant, then Fig. 1 shows that one of the three temperature profiles (denoted by points 1, 2, 3) on the rod is possible. The same situation is also observed under the second-kind boundary conditions on the heated end (regimes 1′, 2′, 3′).

With the above restrictions, the problem of determining a stable steady-state boiling regime is identical to the problem on the stability of steady distri-

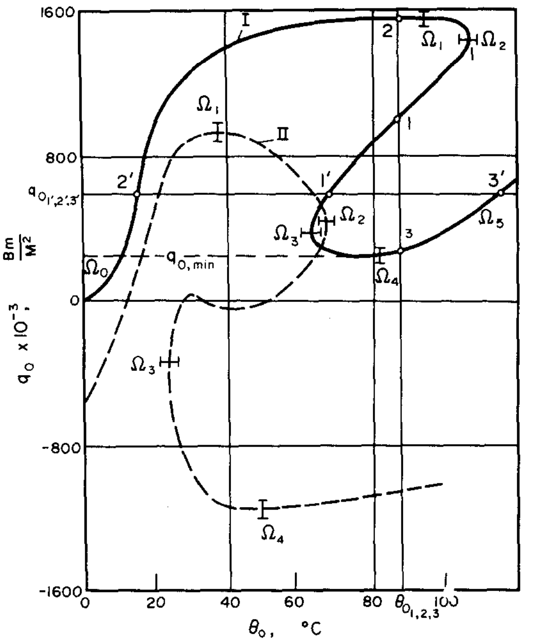

FIG. 1. Dependence of the heat flux q_0 in the heated rod end ($z = 0$) on the local excess temperature θ_0. Dashed curve is for the internal heat source $q_v = 7.10^4$ W/m^2.

bution of the heating wall temperature to the disturbances introduced from outside. With infinitesimal disturbances we can avail ourselves of the already developed methods of the mathematical stability theory and of the small disturbances technique, in particular, as has been done by the present authors in [11]. But if the temperature disturbances cannot be regarded as infinitely small, then this case has no ready solution. The problems of stability "in the great", i.e. to finite deviations, have been developed only for ordinary differential equations with the use of Liapunov's second approach which is based on construction of a special functional. Its properties and the available proven theorems allow certain conclusions on the solution stability. However, as regards the problems of the mathematical physics, including (1)–(4) in the present paper, this theory has not been developed as yet. Therefore, the above problem requires new independent techniques for its solution.

3. TECHNIQUE OF STUDYING STABILITY TO FINITE DISTURBANCES

With the magnitude of disturbances assumed not to be infinitely small, linearization of the boiling curve, $q(\theta)$, in equation (1) is impossible. Let us consider initially the first boundary-value problem, i.e. equation (1) with boundary conditions

$$\theta(z = 0) = \theta_0 = \text{constant} \tag{5}$$

$$\theta(z = L) = \theta_L = \text{constant}. \tag{6}$$

We are in a position to formulate the variational principle which is equivalent to this problem. Let us specify for the temperature θ some virtual displacement $\delta\theta$ consistent with the boundary conditions. Then displacement at the boundary will satisfy the conditions

$$\delta\theta|_{z=0} = \delta\theta|_{z=L} = 0. \tag{7}$$

Consider the integral

$$\int_0^L \left\{\frac{d^2\theta}{dz^2} - \beta[q(\theta) - q_v]\right\}\delta\theta\, dz. \tag{8}$$

The following transformations are valid

$$\int_0^L \delta\left[\frac{1}{2}\left(\frac{d\theta}{dz}\right)^2\right] dz = \int_0^L \frac{d\theta}{dz}\delta\frac{d\theta}{dz}dz = \frac{d\theta}{dz}\delta\theta\Big|_0^L - \int_0^L \frac{d^2\theta}{dz^2}\delta\theta\, dz. \tag{9}$$

Subject to boundary conditions (5) and (6)

$$\frac{d\theta}{dz}\delta\theta\Big|_0^L = 0. \tag{10}$$

Further

$$\int_0^L \beta[q(\theta) - q_v]\delta\theta\, dz = \int_0^L \delta\left\{\int_0^\theta \beta[q(\xi) - q_v]\, d\xi\right\} dz. \tag{11}$$

Thereafter the integral (8) will take on the form

$$\int_0^L -\left\{\delta\left[\frac{1}{2}\left(\frac{d\theta}{dz}\right)^2\right] + \delta\int_0^\theta \beta[q(\xi) - q_v]\, d\xi\right\} dz = -\delta J(\theta) \tag{12}$$

in which

$$J[\theta(z)] = \int_0^L \left\{\frac{1}{2}\left(\frac{d\theta}{dz}\right)^2 + \int_0^\theta \beta[q(\xi) - q_v]\, d\xi\right\} dz \tag{13}$$

is the functional determined in the set of functions $\theta(z)$.

Now we consider the second boundary-value problem, i.e. equation (1) with the boundary conditions

$$\lambda\frac{d\theta}{dz}\Big|_{z=0} = q_0 = \text{constant} \tag{14}$$

$$-\lambda\frac{d\theta}{dz}\Big|_{z=L} = q_L = \text{constant}. \tag{15}$$

For the variational principle we consider the virtual displacement $\delta\theta$, consistent with conditions (14) and (15), i.e. the one which satisfies the equality

$$\delta\frac{d\theta}{dz}\Big|_{z=0} = \delta\frac{d\theta}{dz}\Big|_{z=L} = 0, \tag{16}$$

or

$$\frac{d}{dz}\delta\theta\Big|_{z=0} = \frac{d}{dz}\delta\theta\Big|_{z=L} = 0. \tag{17}$$

In this case the integral (8) is transformed in a somewhat different way. Let us perform the following transformations

$$\int_0^L \delta\left[\frac{1}{2}\left(\frac{d\theta}{dz}\right)^2\right]dz = \int_0^L \frac{d\theta}{dz}\delta\frac{d\theta}{dz}dz = \frac{d\theta}{dz}\delta\theta\Big|_0^L - \int_0^L \frac{d^2\theta}{dz^2}\delta\theta\,dz, \tag{18}$$

$$\int_0^L \frac{d^2\theta}{dz^2}\delta\theta\,dz = -\int_0^L \delta\left[\frac{1}{2}\left(\frac{d\theta}{dz}\right)^2\right]dz + \frac{1}{\lambda}q_L\delta\theta_L - \frac{1}{\lambda}q_0\delta\theta_0, \tag{19}$$

$$\frac{1}{\lambda}q_L\delta\theta_L = \delta\int_0^L \frac{1}{l\lambda}\int_{\theta_L}^{\theta_{L_{\text{dist.}}}} q_L(\theta_L)\,d\theta_L\,dz, \tag{20}$$

$$\frac{1}{\lambda}q_0\delta\theta_0 = \delta\int_0^L \frac{1}{l\lambda}\int_{\theta_0}^{\theta_{0_{\text{dist.}}}} q_0(\theta_0)\,d\theta_0\,dz \tag{21}$$

where $\theta_{L_{\text{dist.}}}$ and $\theta_{0_{\text{dist.}}}$ are the temperatures on the rod ends, $z = L$ and $z = 0$, upon introduction of the disturbance $\delta\theta$.

Besides,

$$\int_0^L \beta[q(\theta)-q_v]\delta\theta\,dz = \int_0^L \delta\int_0^\theta \beta[q(\theta)-q_v]\delta\theta\,dz. \tag{22}$$

With account of what has been said above, the functional (8) for the second boundary-value problem will be of the form

$$J[\theta(z)] = \int_0^L\left\{\frac{1}{2}\left(\frac{d\theta}{dz}\right)^2 + \int_0^\theta \beta[q(\xi)-q_v]\,d\xi\right\}dz - \frac{d\theta}{dz}\Big|_{z=L}(\theta_{L_{\text{dist.}}}-\theta_l) + \frac{d\theta}{dz}\Big|_{z=0}(\theta_{0_{\text{dist.}}}-\theta_0). \tag{23}$$

It is easily seen that the extreme values of $J(\theta)$, if any, may occur only in the solutions of steady-state equation (1) and their stability is to be subjected to the analysis. While the functional behaviour in the vicinity of its extreme values or, in other words, near its stationary points gives evidence on the stability of the steady-state solutions.

In fact, let $\theta(z, \tau)$ be the solution to equation (1) near the steady-state solution. Consider the derivative $dJ/d\tau$ (the first boundary-value problem) with τ assumed as a parameter

$$\frac{dJ}{d\tau} = \int_0^L\left\{\frac{d\theta}{dz}\frac{d^2\theta}{dz\,d\tau} + \beta[q(\theta)-q_v]\frac{d\theta}{d\tau}\right\}dz = \int_0^L \frac{d\theta}{dz}d\frac{d\theta}{d\tau} + \int_0^L \beta[q(\theta)-q_v]\frac{d\theta}{d\tau}dz$$

$$= \frac{d\theta}{d\tau}\frac{d\theta}{dz}\Big|_0^L - \int_0^L \frac{d\theta}{d\tau}\frac{d^2\theta}{dz^2}dz + \int_0^L \beta[q(\theta)-q_v]\frac{d\theta}{d\tau}dz. \tag{24}$$

However, according to (5) and (6),

$$\frac{d\theta}{d\tau}\Big|_L = \frac{d\theta}{d\tau}\Big|_0 = 0.$$

Therefore

$$\frac{d\theta}{d\tau}\frac{d\theta}{dz}\Big|_0^L = 0. \tag{25}$$

Then

$$\frac{dJ}{d\tau} = -\int_0^L \frac{d\theta}{d\tau}\left\{\frac{d^2\theta}{dz^2} - \beta[q(\theta)-q_v]\right\}dz = -\int_0^L\left(\frac{d\theta}{d\tau}\right)^2 dz \leqslant 0. \tag{26}$$

In a similar way we obtain for the second boundary-value problem [with boundary conditions (14) and (15)]

$$\frac{dJ}{d\tau} = \frac{d\theta}{d\tau}\frac{d\theta}{dz}\Big|_0^L - \int_0^L \frac{d\theta}{d\tau}\frac{d^2\theta}{dz^2}dz + \int_0^L \beta[q(\theta)-q_v]\frac{d\theta}{d\tau}dz - \frac{d\theta}{dz}\Big|_{z=L} \times \frac{d\theta_L}{d\tau} + \frac{d\theta}{dz}\Big|_{z=0}\cdot\frac{d\theta_0}{d\tau}\cdot = -\int_0^L\left(\frac{d\theta}{d\tau}\right)^2\cdot dz \leqslant 0. \tag{27}$$

It follows from (26) and (27) that in the solutions of equation (1), with the corresponding boundary conditions, the functional (13) or (23) changes with respect to time in such a way that its value decreases, i.e. $J[\theta(z)]$ tends to minimum. That the functional could decrease about the stable steady-state solutions (points 2 and 3 in Fig. 1), it is imperative that its value in these very points be minimum. For the same reason, the unstable regime (point 1 in Fig. 1) should be either maximum or minimax. The fact that the required Legendre conditions for the maximum [12] are not satisfied at the point 1, indicates that this point is the point of minimax. In this way one can obtain the form of the surface which is determined by the values of the functional. In a three-dimensional space (provided the functional $J[\theta(z)]$ is reduced to the integral $J(p_1, p_2)$ and the disturbances are characterized by two generalized coordinates p_1 and p_2) the surface has the form as shown in Fig. 2. The domains of "attraction" of points 2 and 3 are valleys divided by the crest which we shall call "a watershed" and denote by a dashed-dotted line in Fig. 2. On this crest, point 1 of the minimax is the point of inflection. Transition from point 2 into the "attraction domain" of point 3 is possible only through the "watershed" in any of its points, i.e. by imparting to the initial profile a disturbance having a corresponding value of J not less than that at the "watershed".

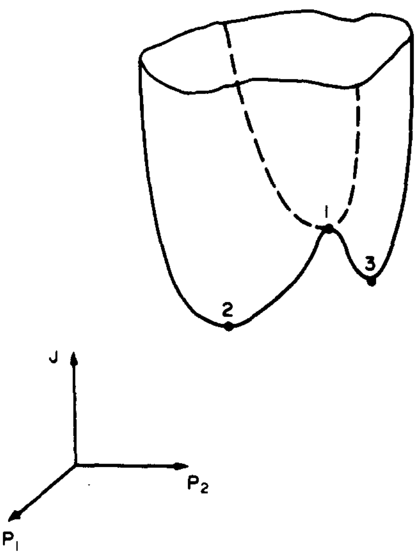

FIG. 2. Qualitative representation of the functional surface form in the system of generalized coordinates p_1 and p_2.

On the basis of the foregoing discussion, the analysis of the boiling heat-transfer stability to the finite disturbances is to be performed in the following succession:

1. The stability of the steady-state profile 2 (or 3) to any chosen type of disturbance is to be considered.

2. The analysed type of disturbance is to be given as a one-parameter family of functions involving the parameter p. When p increases continuously from zero, which corresponds to point 2, the surface of the functional J has a curve which is circumscribed from point 2 (or 3) and which either intersects the "watershed" or not.

3. Profile 2 (or 3) will be stable to the studied type of disturbances until, with increasing p, the "watershed" is passed, which can be expressed by

$$J(\theta_{\text{dist.}}) < J_{w.sh.} \tag{28}$$

4. Since at point 1 (or 1′), i.e. at the inflection point, the minimum of all the $J_{w.sh.}$ values is achieved, inequality (28) can be *a priori* replaced by a more severe one

$$J(\theta_{\text{dist.}}) < J(1) \tag{29}$$

or

$$J(\theta_{\text{dist.}}) > J(1'). \tag{30}$$

Thus, whatever the type of disturbances, fulfilment of (29), provided the quantity $J(\theta_{\text{dist.}})$ has not passed its maximum, is a sufficient condition for the stability of a considered steady-state profile to the one-parameter class of disturbance functions. The calculation procedure should, therefore, comprise the following stages:

(a) numerical evaluation of the steady-state values of the functional for the given boundary-value problem and derivation of their corresponding dependence on the boundary conditions;

(b) choice of the class of disturbances;

(c) calculation and construction of $J(\theta_{\text{dist.}})$ in this class as a function of the parameter p;

(d) determination of a guaranteed safe disturbance based on comparison of $J(\theta_{\text{dist.}})$ with $J(1)$.

4. CALCULATION OF GUARANTEED SAFE DISTURBANCES

Using the method obtained above in Section 3, a study has been made of the heat-transfer stability to both infinitesimal and finite temperature deviations for the case of liquid boiling on the rod. Calculations are performed for Freon-113 and the copper rod (0.006 m in diameter and 0.03 m long, the free end adiabatically insulated) under the boundary conditions of the first, second and the third kind on the fixed end.

Employing the Runge–Kutta method, sixty various steady-state regimes $\bar{\theta}(z)$ have been calculated for $q_v = 0$, which have given an S-shaped curve at $q_0 - \theta_0$ shown in Fig. 1.

Here, in Fig. 1, by dashes an S-shaped curve is plotted for the case when the internal heat sources are present. Their power, q_v, per unit heat releasing surface of the rod is 7×10^4 W/m^2. As is clear from Fig. 1, with the presence of q_v the S-curve undergoes a considerable deformation and displaces downwards. The points Ω_1, Ω_2, Ω_3, Ω_4, are the boundaries of stability in the small, i.e. the boundaries at which a change occurs in the sign of the minimum eigenvalue μ_0 [11] if one follows the S-curve. Thus, for example, instability of the regimes under boundary conditions of the first kind ($\mu_0 < 0$) is observed on the segment $\Omega_2\Omega_3$ of the curve, while under the boundary conditions of the second kind, on the segment $\Omega_1\Omega_4$. As is seen from Fig. 1, in the presence of the internal heat sources q_v the boundaries of stability under the first and the second kind boundary conditions Ω_i still remain at the same points of the S-curve just like in the case of $q_v = 0$.

Let us consider now the case of the boundary conditions of the first kind in the fixed end of the rod provided that $q_v = 0$. For every point of the S-shaped curve the integral (13) has been calculated which represents a steady-state value of the functional for the given boundary conditions (i.e. for the given θ_0) and is plotted in Fig. 3 vs θ_0. For several regimes the curve has indications of the temperature of the free rod end, θ_L. In the range of the temperature heads, θ_0, from 62.5 to 108°C, multiplicity (triplicity) of J is observed which is caused by the multiplicity of the steady-state solutions to the problems (1)–(4) in the same range of θ_0 (see Fig. 1). On the segment $\Omega_2\Omega_3$, the integral at every point has the maximum value as compared with the segments $\Omega_0\Omega_2$ and $\Omega_3\Omega_5$ for the same θ_0. This is in conformity with what has been said about the functional (13) acquiring the minimum values in the regimes stable to infinitesimal disturbances (points on the segments $\Omega_0\Omega_2$ and $\Omega_3\Omega_5$) and the maximum values, in unstable regimes (segment $\Omega_2\Omega_3$).

Likewise, calculation of the integral (23) has been made for the boundary conditions of the second kind. In this case, for the fixed end of the rod (at $q_v = 0$) the

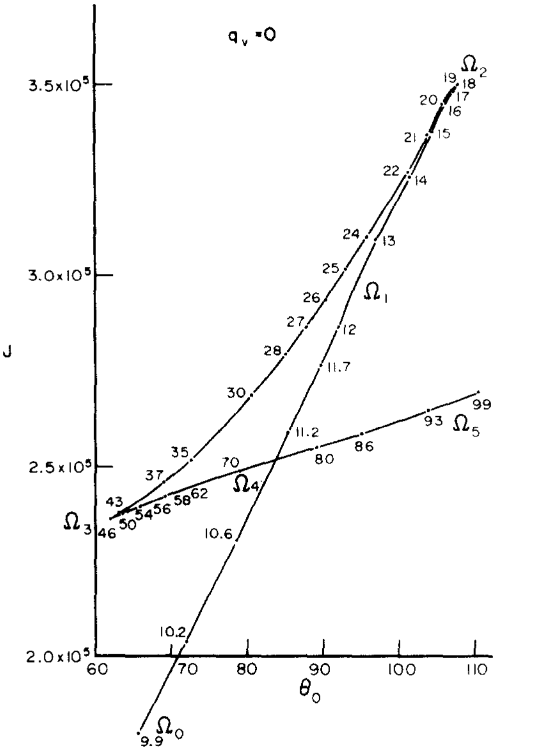

FIG. 3. Stationary values of the functional (13) depending on θ_0 at $q_v = 0$ (first kind boundary conditions).

domain of multiplicity on the S-curve is located with respect to q_0/λ between 600 and 4100°C/m, i.e. on the segment $\Omega_1\Omega_4$. The steady-state values of the integral (23) in this case are presented in Fig. 4 as a function of the boundary conditions q_0/λ. Here again correspondence in the values of the functional is observed for steady and unsteady regimes with the same boundary conditions: J on the segment $\Omega_1\Omega_4$ is larger than that on the segments $\Omega_0\Omega_1$ and $\Omega_4\Omega_5$ for the given q_0/λ.

With internal heat sources in the rod ($q_v = 7 \times 10^5$ W/m^2) with the boundary conditions of the first kind, the dependence of J on θ_0 is of the form presented in Fig. 5. However, under the boundary conditions of the second kind, a quintuple (with respect to q_0) domain appears on the S-curve in Fig. 1. Accordingly, a quintuple domain also reveals itself on the J vs q_0/λ diagram in the form of an additional loop shown separately in Fig. 6 in a larger scale.

In accordance with items (c) and (d) of the above-outlined technique, let us determine the values of the guaranteed safe disturbances which are compatible with the boundary conditions of the second kind, Fig. 7. For their characteristic we shall take the temperature deviation, $\delta\theta$, from the steady-state profile $\theta(z)$ which is uniform at every point along the rod.

With formulae (13) and (23), a calculation is made of the functional of the disturbed profile when the characteristic of the latter is increased. Respective plots are given in Fig. 8. According to the derived stability conditions (30), the safe disturbances (in the sense of transition into another steady state) are characterized by $J(1')$ which should not be exceeded by $J(\theta_{\text{dist.}})$. In the figures, the horizontal dashed lines are plotted for $J(1')$ under the boundary conditions of the second kind. Its abscissa corresponds to the value of the guaranteed safe disturbance.

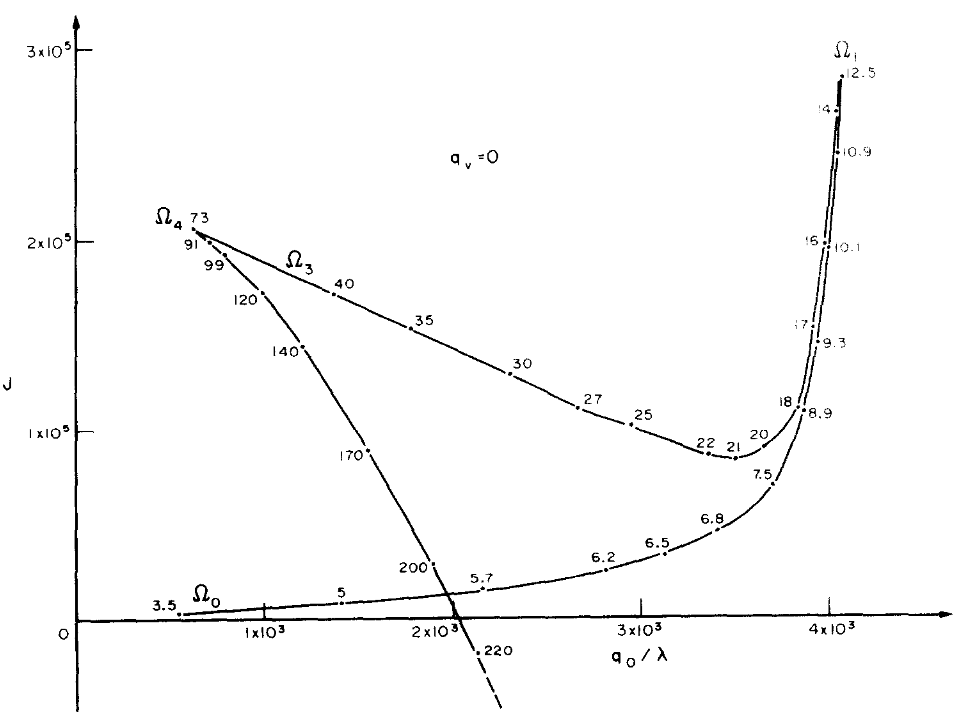

FIG. 4. Stationary values of the functional (23) depending on q_0/λ at $q_v = 0$ (second kind boundary conditions).

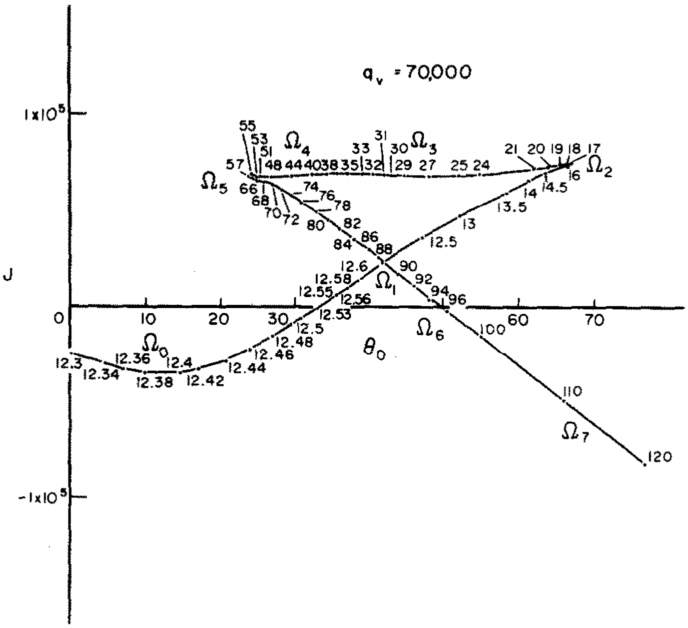

FIG. 5. Stationary values of the functional (13) depending on θ_0 at $q_v = 7 \times 10^4$ W/m^2 (first kind boundary conditions).

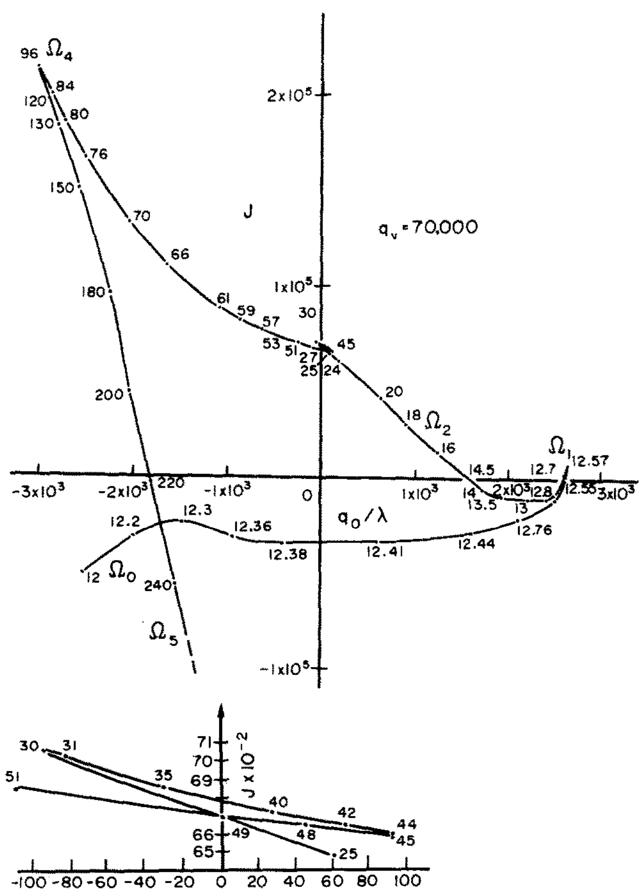

FIG. 6. Stationary values of the functional (23) depending on q_0/λ at $q_v = 7 \times 10^4$ W/m^2 (second kind boundary conditions).

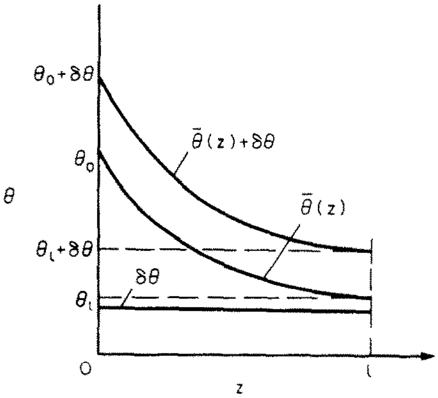

FIG. 7. Disturbance of the stationary temperature profile comparable with the second kind boundary conditions.

replaced by another) is determined by the distribution function for the normal law

$$F(\delta\theta_{cr}) = 0.5+\phi\left(\frac{\delta\theta_{cr}-M}{\sigma}\right), \tag{32}$$

where

$$\phi(u) = \frac{1}{(\sqrt{2\pi})^{1/2}}\int_0^u \exp(-t^2/2)\,dt$$

is the Laplace function. The disturbances exceeding the critical ones are observed with the probability

$$1-F(\delta\theta_{cr}) = 0.5-\phi\left(\frac{\delta\theta_{cr}-M}{\sigma}\right). \tag{33}$$

Let us divide the time of the setup operation, τ_{op}, into N equal intervals $\Delta\tau = \tau_{op}/N$, and denote the number of

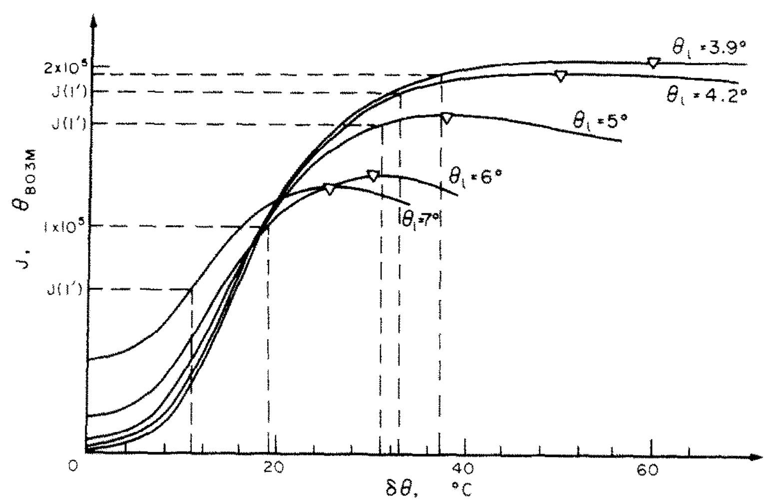

FIG. 8. Change of the functional with increase in the disturbance characteristic (second kind boundary conditions).

5. ILLUSTRATION OF THE USE OF THE DEVELOPED TECHNIQUE

Employment of the functional method allows a sufficiently simple evaluation of the rated heat flux which provides the required reliability of the setup operation.

Suppose there is a certain steady-state profile $\bar\theta(z)$ into which disturbances $\theta_{\text{dist.}} = \delta\theta\varphi(z)$ are introduced at times $\tau_1, \tau_2, \ldots$ during the setup operation. The amplitude of the disturbances $\delta\theta$ is of a probability character. Depending on the operating characteristics of the setup, there can be different laws governing distribution of disturbances with respect to the amplitude. Let the distribution obey the normal law with the probability density

$$f(\delta\theta) = \frac{1}{\sigma(\sqrt{2\pi})^{1/2}}\exp\left[-\frac{1}{2}\left(\frac{\delta\theta-M}{\sigma}\right)^2\right],$$
$$-\infty < \delta\theta < \infty \tag{31}$$

where M is the mathematical expectation and σ is the root-mean-square deviation of the random quantity $\delta\theta$. The probability of this random quantity being less than the critical value, $\delta\theta_{cr}$ (at which one regime is

the finite disturbances for the period $\tau_i+\tau_{i+1}$ by m_i. Then, the concept of the temperature field disturbance flow parameter can be introduced as

$$\chi(\tau) = \frac{m(\tau, \tau+\Delta\tau)}{\Delta\tau}. \tag{34}$$

The stable operating conditions allow the assumption that for the flow of disturbances the following conditions are satisfied:

(a) The parameter of the flow of disturbances is constant quantity, χ = constant.

(b) The number of disturbances in the considered period is not associated with the number of disturbances in the preceding interval of time.

(c) Two or several disturbances at a time are impossible.

With the fulfilment of the above requirements, the probability of "n" events is described by the Poisson law

$$P(n, \Delta\tau) = \frac{(\chi\tau)^n}{n!}\exp(-\chi\tau), \tag{35}$$

where $n = 0, 1, 2, \ldots, \infty$.

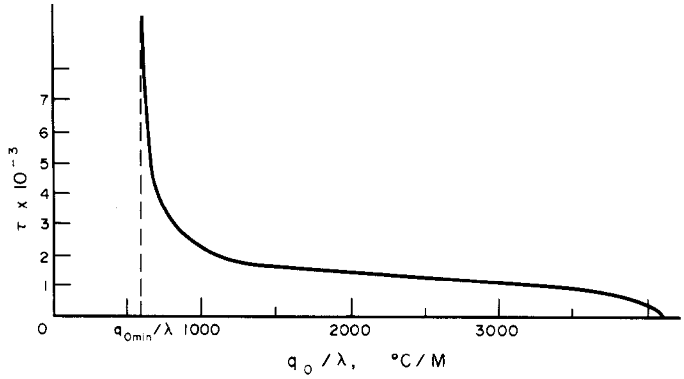

FIG. 9. Time of safe operation of the system vs the heat flux density.

When $n = 0$, i.e. the disturbances for the time τ are absent, the probability of this event is given by

$$P(\tau) = \exp(-\chi\tau). \qquad (36)$$

Using the law of reliability, we can determine the mean time of operation between the disturbances

$$\Delta\bar{\tau} = \int_0^\infty \tau\chi\exp(-\chi\tau)\,d\tau$$

$$= -\tau\exp(-\chi\tau)|_0^\infty + \int_0^\infty \exp(-\chi\tau)\,d\tau = \frac{1}{\chi}. \qquad (37)$$

This time seems to be connected with the above distribution of disturbances with respect to their amplitudes. Therefore, the final life-time of the system is

$$\tau_{l.t.} = \frac{1}{\chi[1 - F(\delta\theta_{cr})]}. \qquad (38)$$

As an example, let us estimate the life-time of the above system consisting of the copper rod and Freon-113 with the disturbances as shown in Fig. 7. Suppose the parameter of the temperature profile disturbances flow, χ, is equal to $10^{-3}\,h^{-1}$, the RMS deviation σ of a random value of the disturbance is 30°C. Then, using the functional in the way as outlined at the end of Section 3, we shall find the values of $\delta\theta_{cr}$, corresponding to different heat flux densities q_o/λ in the fixed end of the rod. Thereafter, with the help of formulae (38) and (33) and the table of the Laplace function we determine the life-time of the setup for each of the chosen heat fluxes. The results are presented in Fig. 9. The diagram demonstrates that, for example, for the heat flux close to the maximum, $q_0 = 1\,240\,000\,W/m^2$ ($q_0/\lambda = 3300$°C/m), the safe life-time is 1000 h. With a decrease in the heat flux to $q_0 = 263\,000\,W/m^2$ ($q_0/\lambda = 700$°C/m), the safe life-time increases up to 4000 h.

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CALCUL DE LA STABILITE DE L'EBULLITION EN RESERVOIR A DES PERTURBATIONS FINIES

Résumé—Une étude théorique de la stabilité de l'ébulliton en réservoir est faite pour une tige à surface non isotherme. La formulation mathématique du problème est fondée sur une approche thermique par laquelle l'étude de la stabilité est basée sur la solution du problème de la conduction thermique dans une tige avec des conditions aux limites de première et de seconde espèces. On analyse la stabilité à des perturbations de température. Par application d'un principe varationel, une mesure de stabilité est donnée par une fonctionnelle spéciale.

BERECHNUNG DER STABILITÄT DES WÄRMEÜBERGANGS BEIM BEHÄLTERSIEDEN GEGENÜBER ENDLICHEN STÖRUNGEN

Zusammenfassung—Es wurde eine theoretische Studie über die Stabilität des Wärmeübergangs beim Behältersieden an der nicht-isothermen Oberfläche eines Stabs durchgeführt. Die mathematische Formulierung des Problems basiert auf dem thermodynamischen Vorgehen, welches erlaubt, die Stabilitätsuntersuchung auf die Lösung des Randwertproblems der Wärmeleitung für einen Stab mit Randbedingungen erster und zweiter Art zurückzuführen. Die Stabilität bei endlichen Temperaturabweichungen wurde analysiert. Mit Hilfe eines Variationsprinzips wurde als Maß für die Stabilität ein speziell gebildetes Funktional gefunden. Die vorgeschlagene Methode wird durch die Berechnung der sicheren Temperaturstörung erläutert.

РАСЧЕТ УСТОЙЧИВОСТИ ТЕПЛООБМЕНА ПРИ КИПЕНИИ ЖИДКОСТИ В БОЛЬШОМ ОБЪЕМЕ К ВОЗМУЩЕНИЯМ КОНЕЧНОЙ ВЕЛИЧИНЫ

Аннотация — Проведено теоретическое исследование устойчивости теплообмена при кипении жидкости в большом объеме на неизотермической поверхности стержня. Математическая формулировка задачи осуществлена с позиций термического подхода, в результате чего исследованию на устойчивость подвергается решение краевой задачи теплопроводности для стержня с граничными условиями 1-го и 2-го рода. Рассмотрена устойчивость к температурным отклонениям конечной величины. С помощью вариационного принципа найдена мера устойчивости — специально построенный функционал. Иллюстрацией предложенного метода служит расчет величины безопасного возмущения температуры.